# About the Next Topic and Lessons 18-21:
## Applications of the Derivative: Intervals for Increasing and Decreasing Functions, Minima, Maxima, Concavities and Points of Inflection

Let $x_0$ be a zero of the first derivative. The slopes of the tangent lines at points immediately before and after the point $x_0$ are given by the first derivatives.

**Uses of the First Derivative Test** (We will use a sign diagram)

**1.** To determine the **intervals** for increasing and decreasing functions.
Here, we find the first derivative, equate it to zero and solve to obtain critical values, which will be used on a sign diagram for testing. (see. p.143)

**2.** To find the r**elative minima and maxima.**
Here also, we find the first derivative, equate it to zero and solve to obtain critical values to be used on a sign diagram for testing.(see p151, 153)

**3**. To find the **absolute** minima and absolute maxima of functions.(see p.158)
We find the first derivative, set it to zero and solve to obtain critical values which will be used to evaluate the original functional equation. We also evaluate the functional equation for the end points of the domain of the function. The minimum and maximum values may occur at the end points.

**4.** For **applied minima** and minima problems. (see p.163)

**5.** To find i**nflection points**: Let $x_0$ be a critical $x$-value of $f$. On a sign diagram, if the sign of $f'(x)$ immediately to the left of $x_0$ is the same as the sign of $f'(x)$ immediately to the right of $x_0$, then $x_0$ is at an inflection point.

**6.** To find **concavity.** (see p.152 for some description of " $f'(x)$ increasing")
If $f'(x)$ is increasing on an interval, then $f$ is concave up on that interval
If $f'(x)$ is decreasing on an interval, then $f$ is concave down on that interval.

**Uses of the Second Derivative Test**

**1.** To find the **relative minima** and maxima.
Here, we find the first derivative, set it to zero and solve to obtain critical values. We then find the second derivative and then evaluate the second derivative using the critical values from the first derivative. For a critical $x_0$,
if $f''(x_0) > 0$ then $f(x_0)$ is a relative minimum; (curve is concave up)
if $f''(x_0) < 0$, $f(x_0)$ is a relative maximum; (curve is concave down) but
if $f''(x_0) = 0$ or is infinite, use the **first derivative** test.

**2**. To determine **concavities** and points of **inflection**.
Find the first derivative, and find the second derivative. Set the second derivative to zero and solve to obtain values tfor testing intervals for concavity and inflection points, using a sign diagram similar to the one used in the first derivative test. Concavity is determined by the sign of the second derivative.

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| Let $x_{02}$ be a zero of the second derivative; $x_{02+}=$ to the right of $x_{02}$; $x_{02-}=$ to its left. |
| If $f''(x_{02+}) > 0$ in $(a,b)$, $f$ is concave up in $(a,b)$, but concave down if $f''(x_{02+}) < 0$ |
| If $f''(x_{02-}) > 0$ in $(a,b)$, $f$ is concave up in $(a,b)$, but concave down If $f''(x_{02-}) < 0$ |

**3.** To determine i**nflection points** The point $(x_{02}, f(x_{02}))$ is an inflection point of $f$ if $f''(x_{02}) = 0$ or is undefined and $f''(x)$ changes sign as $x$ increases through $x_{02}$ or $f'''(x_{02}) \neq 0$ if $f'''(x_{02})$ exists. **On a sign diagram, an inflection point is at $x_{02}$ where the second derivative changes sign.**